JUDGMENTS OF 3D BARS IN DEPTH

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Twenty participants judged the relative size of two bars portrayed at different locations in a 3D bar graph. Bars were co-located, placed side by side (near adjacent), or the small bar occluded the large (near or far occluded). Error was greater for the far occluded condition and there was greater variability in bias scores. To account for observed error, we proposed a model that distinguishes between cyclical bias commonly observed in proportion judgments and bias resulting from improper size-distance scaling. The model was fit to data, and the results indicated that the absolute value of size-distance scaling parameter γ was greater in the far occluded condition. Inclusion of γ increased R^2 for far and near occluded conditions only. Bar location did not affect cyclical bias. Thus, judgments of the relative sizes of bars in 3D bar graphs showed increased error when the bars were separated, due to inaccurate size-distance scaling.

INTRODUCTION

In an attempt to maximize the amount of information available at a glance, modern information visualization systems make use of novel display techniques in which 3D graphics are heavily relied upon. In such systems and elsewhere, three-dimensional (3D) bar graphs are increasingly used to display complex data of all kinds, and 3D bars are commonly portrayed at different distances from the observer. For example, this occurs when bars are placed at different locations on a map or terrain surface viewed at an oblique angle. It also occurs with 3D bar graphs or histograms where two axes code variables and bar height is used to represent a third variable. However, the use of a 3D representation introduces the potential for error in comparisons of the elements presented at different depths. In this paper, we address this issue.

Depicting 3D bars in depth using an orthographic representation can lead to more distant bars appearing larger than they actually are (Wickens & Hollands, 2000). A perspective rendering offers a solution by portraying more distant objects at smaller sizes, but there may be insufficient depth information to specify precisely the depths of objects and therefore size-distance scaling may be inaccurate (Holway & Boring, 1941; McKee & Smallman, 1998).

More specifically, retinal size (visual angle) is determined by an object's size and the viewing distance (Hershenson, 1999). In real-world scenes, the variety of redundant depth information available leads to the phenomenon of *size constancy*: perceived size is independent of retinal size and does not change with viewing distance (Holway & Boring, 1941; McKee & Smallman, 1998). In these situations, size-distance scaling is said to be accurate. However, in situations where the number and kind of available depth cues are reduced, retinal size affects perceived size, because the distance of the judged object is not perceived accurately (Holway & Boring, 1941). Perceived size, therefore, will be affected by available depth information and size constancy may not hold.

For 3D displays, depth cues can be added to allow the sizes of objects at different distances to be judged more accurately, but the contribution of various depth cues is not well understood (Elliott et al., 1996) and varies with viewing distance (Surdick et al., 1994) and across individuals (Westerman & Cribbin, 1998). Our experimental question, therefore, was: Does size-distance scaling allow accurate judgments of portrayed size in 3D bar graphs given the depth cues typically available (linear perspective, texture, relative height and occlusion)? The issue is not limited to 3D bar graphs: height is similarly used in 3D geographic map displays to indicate altitude of terrain, and in 3D command and control or air-traffic control systems to depict the height of aircraft. Judgments of these heights may similarly show poor size-distance scaling.

Proportion Judgments

The *proportion judgment* has received little scrutiny in the size-distance scaling context. However, it is a common graph reading task (Hollands & Spence, 1992; 1998) and should allow the investigation of size-distance scaling if the quantities being compared are placed at different distances. Examine Figure 1 and ask yourself: what proportion is the small bar of the larger? When the two bars are portrayed at different distances, the estimated proportion should reflect size-distance scaling mechanisms or the judgment will not be accurate.

The cyclical power model (CPM) (Hollands & Dyre, 2000; Spence,1990) offers a technique for understanding the sources of error in proportion judgments. In 3D bar graphs, geometric volume represents the magnitudes of underlying quantities. Unfortunately, human judgments of volume are biased (Hollands & Dyre, 2000; Stevens, 1975; Wickens & Hollands, 2000). Data obtained by Spence (1990) and re-analyzed by Hollands and Dyre show cyclical bias patterns when observers make proportion judgments with 3D bars. According to the CPM, the Stevens exponent for the stimulus continuum determines the amplitude of the cyclical pattern Using the magnitude estimation procedure, exponents for volume, area, and length are approximately 0.6, 0.8, and 1 (Stevens, 1975). Using the CPM Hollands and Dyre found similar exponents for judgments of proportion with 3D bar graphs (β = 0.6-0.8). Observers can typically judge length (bar height) or area (face of bar) in 3D graphs since the third

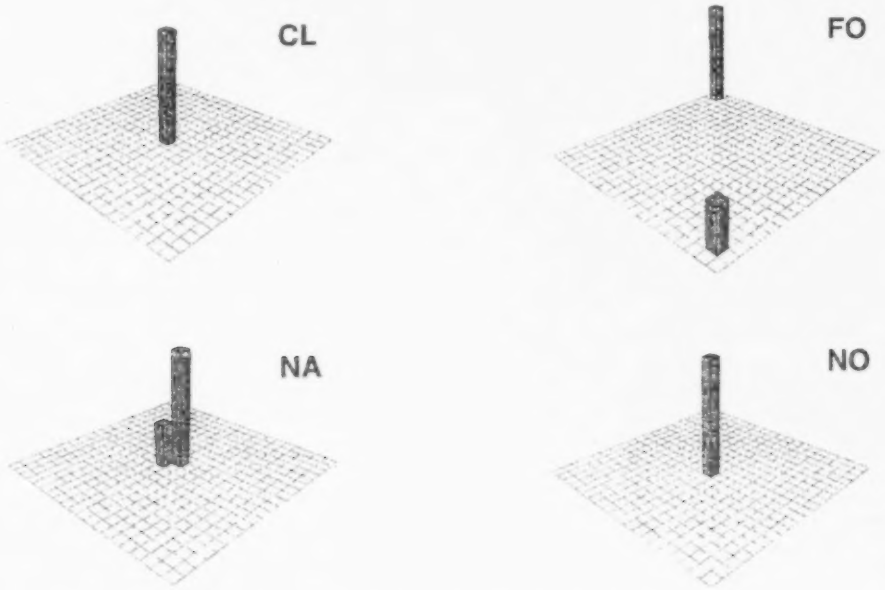

Figure 1. *Examples of stimuli used in the experiment (top: co-located, far-occluded; bottom: near-adjacent, near occluded).*

dimension is not used to portray data values. This may explain why exponents obtained with 3D bar graphs are slightly higher than those obtained using "proper" volume judgments.

According to CPM, the frequency of the bias pattern is determined by the use of intermediate reference points. Most people show a two-cycle pattern (over-under-over-under) with common graph types (bars, pies) (Hollands & Dyre, 1997). The two-cycle model version uses reference points at 0, 50, and 100% (Hollands & Dyre, 2000). However, the one-cycle pattern (over-under) is also relatively common (implying reference points at 0 and 100%). To obtain the best estimate of the Stevens exponent, it is important to choose the best-fitting model version, and hence we fit one- and two-cycle versions of CPM to our data.

A size-distance scaling parameter can be added to the CPM,

$$P' = \gamma + P,$$

where P' represents a subjective proportion rescaled according to the distances of the objects, P represents CPM's predicted proportion, and γ represents the size-distance scaling factor. This allows an assessment of two contributing sources to judgment error: bias that results from the judgment of portrayed volume or area, and bias resulting from inaccurate size-distance scaling.

What happens to proportion estimates when the two bars used for the judgment are portrayed at different distances? If size constancy holds, nothing should change relative to the situation where the bars are portrayed at the same distance (no change in accuracy or bias). If size-distance scaling is not preserved, judged sizes should show a reduced accuracy and biases may result. If observers judge retinal size then this would lead to general overestimation of the proportion or a positive bias (since the "true" size of the rear bar will be underestimated). If observers perceive the far bar as farther than actually depicted, an overconstancy (Teghtsoonian & Teghtsoonian, 1970) would result leading to overestimation of the far bar, underestimation of the proportion, and a negative bias. A third possibility is that size estimates simply become less stable, with both overconstancy and underconstancy (either through judgments of retinal size or otherwise) occurring. In this case, there may be no change in net bias but an increased variability in the scores.

The purpose of the experiment was to examine observers' perceptual biases in proportion judgments with 3D bar graphs where bars were shown at varying locations. Examination of measures of central tendency and dispersion of γ across conditions will provide insight into how size-distance scaling operates with 3D bar graphs. Examination of β will allow us to determine whether the choice of stimulus continuum varies with bar location.

METHOD

Participants

There were 20 participants (10 male, 10 female), employees of a scientific institute. All had normal or corrected-to-normal vision.

Apparatus and Stimuli

Stimuli were presented on a Hitachi Superscan 814

21" CRT monitor at 1280 x 1024 resolution at .31 mm dot pitch controlled by a graphics workstation. Viewing distance was approximately 50 cm.

On each trial, a 3D model was shown, consisting of a pair of bars displayed on a 20 x 20 grid (each square on the grid therefore forms a 1 x 1 unit in the 3D model). The height of the large bar was 10 units; bar length and width were 1 unit each. The viewing angle was 45 degrees from horizontal such that bar tops were visible, and the grid was oriented with one corner closest to the viewer (see Figure 1). The bars were dark gray, the background was white, and the grid lines were black.

There were four bar location conditions: Co-Located (CL), Far Occluded (FO), Near Adjacent (NA), and Near Occluded (NO). In the CL condition, the large bar was 8.2 cm high x 1.30 cm wide. For the FO condition, the large bar was 6.7 cm high and 1.05 cm wide. Figure 1 shows examples of the stimuli. In the occluded conditions, the larger bar was always placed further back (if the smaller bar was at the back, it would not have been visible in some cases).

Using 3D model units, the height of the small bar was defined as a proportion of the large bar. There were 30 bar pair stimuli for each condition, with the same set of 30 proportions across conditions. The set was randomly chosen without replacement from the set of integer values between 1-100 (re-expressed as proportions).

Design and Procedure

The experiment had a within-subjects design with four levels of bar location. The 30 proportions x 4 conditions produced a block of 120 trials. There were 4 blocks containing identical stimuli and the order of trials within each block was randomized. The participant's judged proportion and response time were obtained.

Participants were randomly assigned to a participant number, which determined the random order of trials within each block. Each participant read the instructions and completed a set of practice trials prior to the experimental trials. For each trial, a bar graph with two different sized bars was displayed on a grid. The participant's task was to judge the proportion the small bar was relative to the large bar. The participant responded using the numeric keypad and pressed the space bar to advance to the next trial. The session took approximately 40 minutes to complete.

RESULTS

Bias and Absolute Error

A *bias* score (judged – true proportion) was computed for each trial. Mean values were calculated for each participant in each condition, and submitted to a within-subjects analysis of variance (ANOVA) with bar location as the independent variable. Mean biases were negative: M = -2.07, -4.08, -1.04, and -2.48 for CL, FO, NA, and NO, respectively. However, there were no differences among the means, $F(3,57) = 1.35$, $MSe = 23.70$, $p > .25$. Between-subjects variance was greater in the FO condition ($s^2 = 95.53$) than in other conditions, ($s^2 = 7.51$, 2.35, and 7.38 for CL, NA, and NO, respectively), Hartley's $F_{max}(3,19) = 40.70$, $p < .0001$.

An *absolute error* score (|judged proportion – true

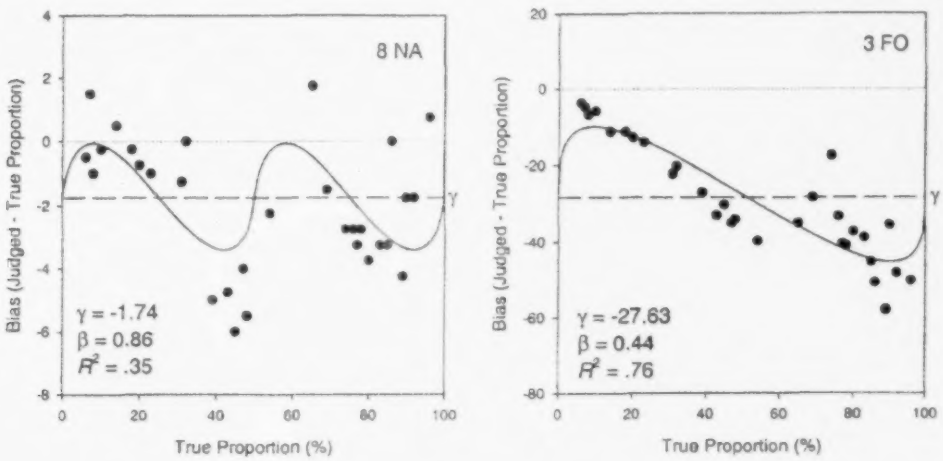

Figure 2. *Example of one- and two-cycle bias patterns. Each graph shows bias as a function of true proportion (expressed as a percentage), and the model fit for the CPM with an added size-distance scaling parameter (γ).*

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proportion]) was computed for each trial. Mean values were submitted to a within-subjects ANOVA with bar location as the independent variable. Absolute error was larger for the FO condition (M = 10.16) than for any other (M = 4.54, 3.72, and 5.02 for CL, NA, and NO, respectively), $F(3,57) = 16.22$, $MSe = 10.48$, $p < .0001$, Newman-Keuls (NK), $p < .05$.

Model Fits

Using a nonlinear estimation procedure, we fit one- and two-cycle versions of the CPM with the γ parameter (see Hollands & Dyre, 2000 for model equations). Figure 2 shows examples of one and two cycle patterns and the fit of the model in each case. The one-cycle pattern was generally more common, with 15,13, 10, and 12 (of 20) participants (CL, FO, NA, and NO conditions, respectively) showing a larger R^2 value for the one- than the two-cycle fit.

Estimated γ values from the best-fitting model version were submitted to a within-subjects ANOVA. Mean γ-values were generally negative (-1.83, -3.66, -0.82, and -2.21 for CL, FO, NA, and NO, respectively), but did not vary as a function of bar location, $F(3,57) = 1.30$, $MSe = 21.32$, $p > .25$. However, between-subjects variance was greater in the FO condition ($s^2 = 90.51$) than in the other conditions, ($s^2 = 6.11$, 2.53, and 7.21 for CL, NA, and NO, respectively), Hartley's $F_{max}(3,19) = 35.73$, $p < .0001$.

The absolute value of each γ value was taken and submitted to a within-subjects ANOVA. The absolute value of γ was larger for the FO condition (M = 7.17) than any other (M = 2.18, 1.38, and 2.83 for CL, NA, and NO, respectively), $F(3,57) = 11.03$, $MSe = 12.13$, $p < .0001$, NK, $p < .05$.

Estimated β-values from the best-fitting model version were submitted to a within-subjects ANOVA. Mean β-values (0.90, 0.80, 0.89, and 0.87 for CL, FO, NA, and NO, respectively) did not vary as a function of bar location, $F(3,57) = 1.54$, $MSe = 0.21$, $p > .20$.

We computed the mean R^2 for predicted bias for CPM versus CPM + γ parameter (best-fitting versions) and found R^2-values of 0.30 and 0.36, respectively. (Note that R^2 values for predicted proportion are typically near unity due to the fact that the model correctly predicts an increase in P with true proportion and so we here report the values for predicted bias to assess differences in fit. See Hollands & Dyre, 2000 on this point). However, we submitted R^2 values for the best-fitting model version for each subject in each condition with and without gamma to a 4 x 2 within-subjects ANOVA (bar location and gamma-in-fit) and found that the advantage of adding γ was only evident in the FO and NO conditions, $F(3,57) = 7.36$, $MSe = 0.0048$, $p < .0005$.

Response Time

A mean response time was computed for each subject in each condition. These values were submitted to a within-subjects ANOVA with bar location as the independent variable. Bar location had no effect on response times, $F(3,57) = 1.21$, $MSe = 1080433$, $p > .30$.

DISCUSSION

The purpose of the experiment was to examine observers' perceptual biases in proportion judgments with 3D bar graphs where bars were shown at varying locations. Although bias was unaffected by bar location, the absolute error of the judgment (and variability of bias) was larger in the far-occluded condition.

When CPM was augmented with a size-distance scaling parameter γ and fit to bias scores, there was greater between-subjects variability in γ for the far-occluded condition than the other conditions. The absolute value of γ was also greater in the far-occluded condition.

In combination, the results indicated that size estimates in 3D bar graphs were less stable when the bars were placed farther apart, with both overconstancy and underconstancy occurring. Presumably, the available depth information does not accurately specify the relative distances of the two objects, and this poor specification leads to imprecise size estimates.

The Stevens exponent (β) remained constant regardless of bar location. This implies that the same perceptual dimension (or combination of dimensions) is judged in all cases. The value of approximately 0.87 is higher than typically obtained from magnitude estimation for judgment, but consistent with estimates obtained in earlier work with pairs of 3D bars portrayed at the same distance (Spence, 1990; Hollands & Dyre, 2000). As in these studies, β may be higher because observers can judge length or area instead of volume.

In addition, we determined that the fit of CPM can be improved with the addition of the size-distance scaling parameter, but that this improvement is limited to conditions where the two bars are portrayed at different distances. This result serves as an indication that modeling the size-distance scaling as an additive parameter captures bias resulting from inappropriate size distance scaling.

In combination, the results suggest that the increased error in the far occluded condition was a result of improper size-distance scaling, and that bar location had no effect on the amplitude of observed cyclical bias.

Implications for Design

An implication for graph design is that bars should be portrayed at similar depths in 3D bar graphs if accurate judgments are necessary, or that 2D graphs (where there is no variation in depth) should be considered as an alternative. 3D bars may be augmented by graduated axes scaled to various depths; research is planned to examine the effect of these augmentations on size-distance scaling.

This experiment illustrates a more general technique in which the effect of different sources of depth information (e.g. motion parallax, stereopsis, etc.) on judgment error can be assessed for 3D information visualization applications. Future work is planned in this area.

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